THERMAL STATE OF DYNAMIC CALORIMETERS

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It is established here how the thermal state of a calorimeter affects the accuracy of power measurements. A mathematical model of a dynamic calorimeter is proposed and its thermal state is analyzed.

In [1-3] the authors have proposed a method and described a new device, a dynamic biocalorimeter, for measuring thermal fluxes variable in time which are generated by energy sources of a diverse nature such as, for example, biological sources.

The thermophysical model is shown in Fig. 1: it consists of a chamber 1, a shell 3, and an insulation 4. A closed thin interlayer of air 2 separates the shell from the chamber.

It will be assumed that the power of the source $P(\tau)$ inside the chamber can vary with time according to any law. The energy generated by the source passes through the chamber walls and heats them up, whereupon it enters the shell and raises its temperature.

Furthermore, the calorimeter may be exposed to various forms of extraneous thermal noise such as, for example, variations in the ambient temperature $t_a(\tau)$ around it, or a thermal flux $q(\tau)$ appearing at the outer surface of the device. Extraneous thermal noise may be regarded as a second cause of variations in the thermal state of the shell and, consequently, of the entire device.

As has been shown in [1], the following equation relates the power $P(\tau)$ of an energy source to the thermal state of the chamber and the shell:

$$P(\tau) = \frac{1}{mF}\frac{dt_{ch}}{d\tau} + \frac{1}{F}\left[t_{ch}(\tau) - t_{sh}(\tau)\right], \quad m = \frac{kS_{ch}}{C_{ch}}, \quad F = \frac{1}{kS_{ch}}. \tag{1}$$

Inasmuch as extraneous noise has been accounted for in Eq. (1), this noise should not affect the results of measurements of power P generated by the source. Consequently, any noise level is theoretically permissible. The error in the power measurement is different, however, at different noise levels and depends on the noise characteristics. In order to demonstrate the validity of this statement, we derive a formula for calculating the error in a power measurement. It will be assumed, moreover, that systematic errors have been either eliminated or taken into account by corresponding correction terms, and that random errors are distributed normally. From Eq. (1) and according to [4] we then have (see Appendix):

$$\frac{\Delta P}{P} = \sqrt{\frac{a + bX^2 + cY^2}{(X + Y)^2}},$$

$$X = t_{ch}(\tau) - t_{sh}(\tau), \quad Y = \frac{\Pi}{m}, \tag{2}$$

$$a = \frac{\Delta t_{ch1} + \Delta t_{ch2}}{m^2(\tau_2 - \tau_1)^2} + \Delta t_{ch}^2 + \Delta t_{sh}^2, \quad b = \left(\frac{\Delta F}{F}\right)^2,$$

$$c = \frac{\Delta\tau_1^2 + \Delta\tau_2^2}{(\tau_2 - \tau_1)^2} + \left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta F}{F}\right)^2, \quad \Pi = \frac{t_{ch2} - t_{ch1}}{\tau_2 - t_1},$$

$$\frac{\Delta m}{m} = \sqrt{\frac{1}{(\ln Q)^2}\left[\left.\frac{\Delta t_{ch}^2 + \Delta t_{sh}^2}{(t_{ch} - t_{sh})^2}\right|_3 + \left.\frac{\Delta t_{ch}^2 + \Delta t_{sh}^2}{(t_{ch} - t_{sh})^2}\right|_4\right] + \frac{\Delta\tau_3^2 + \Delta\tau_4^2}{(\tau_4 - \tau_3)^2}},$$

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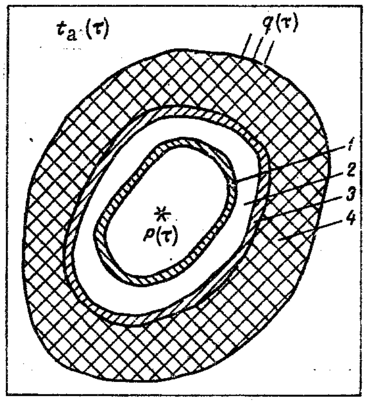

Fig. 1. Thermophysical model of the dynamic calorimeter.

$$\frac{\Delta F}{F} = \sqrt{\left(\frac{\Delta P}{P}\right)_c^2 + \frac{\Delta t_{ch}^2 + \Delta t_{sh}^2}{(t_{ch} - t_{sh})^2}\Bigg|_c},$$

$$Q = \frac{(t_{ch} - t_{sh})_3}{(t_{ch} - t\ )_4}.$$

Parameters a, b, c are only weakly dependent on the thermal state of the chamber and shell so that, to the first approximation, they may be considered constant. A variation in the error $\Delta P/P$ during a test is then related only to the variation in quantities X and Y, i.e., to the variation in the thermal state of the chamber and the shell. The error as a function of the thermal state is shown in Fig. 2, based on Eq. (2) for one of the calorimeters described in [1, 7].

According to Eq. (2), the error $\Delta P/P$ becomes infinite as $X/Y \to (-1)$

$$\lim_{X/Y \to -1} \frac{\Delta P}{P} = \infty.$$

A change of X from 0 to 1 (Fig. 2) at Y = −2, for example, causes the error to increase from 3.5 to 7.0%, but $\Delta P/P = \infty$ when X = 2. The error depends also on the accuracy class of the measuring instruments. The minimum error $\Delta P/P$ = 1.4% in Fig. 2 corresponds to instruments of class 1.0 accuracy. It can be shown, with the aid of Eq. (2), that instruments of class 0.5 accuracy will reduce the error in power measurements to $\Delta P/P$ = 0.8%.

Thus, an analysis of Eq. (2) confirms that the error in power measurements depends on the calorimeter design parameters m and F, on the accuracy class of the instruments, and on the thermal state of both chamber and shell.

We now proceed to analyze the thermal field of the calorimeter. Let us impose here the following constraints: (a) the thermal flux q is uniformly distributed over the insulation surface and (b) the insulation may be regarded as a flat wall.

Let us then formulate the law of energy conservation for the various calorimeter components. The energy $Pd\tau$ coming from the source during the time $d\tau$ both changes the enthalpy of the chamber $C_{ch}dt_{ch}$ and passes on through the gap to the shell $kS_{sh}(t_{ch}-t_{sh})d\tau$, i.e.,

$$P(\tau) = C_{ch}\frac{dt_{ch}}{d\tau} + kS_{ch}(t_{ch} - t_{sh}). \tag{3}$$

The energy $kS_{ch}(t_{ch}-t_{sh})d\tau$, which has passed through the gap, both changes the enthalpy of the shell $C_{sh}dt_{sh}$ and passes on to the insulation $(-\lambda_i(\partial t_i/\partial x)|_{x=0}S_{sh})$, i.e.,

$$kS_{ch}(t_{ch} - t_{sh}) = C_{sh}\frac{dt_{sh}}{d\tau} - \lambda_i \frac{\partial t_i}{\partial x}\Big|_{x=0} S_{sh}. \tag{4}$$

The temperature field of the insulation is described by the Fourier equation, which becomes here

$$\frac{\partial t_i}{\partial \tau} = a_i \frac{\partial^2 t_i}{\partial x^2}. \tag{5}$$

by virtue of assumption (b). The boundary condition at the outer surface of the insulation will be written as follows:

$$-\lambda_i \frac{\partial t_i}{\partial x}\Big|_{x=l} = \alpha(t_i - t_{sh})_{x=l} - q(\tau). \tag{6}$$

The condition

$$t_i|_{x=0} = t_{sh} \tag{7}$$

must be satisfied at the boundary between shell and insulation. With initial conditions added to Eqs. (3)-(7), we have now a mathematical model of the calorimeter. Solving this system of equations for various forms of functions $P(\tau)$, $q(\tau)$, and $t_a(\tau)$, one can find the relations $X(\tau) = t_{ch} - T_{sh}$ and $\Pi(\tau) = dt_{ch}/d\tau$ determining the thermal state of the calorimeter. Such a solution of Eqs. (3)-(7) is fraught with serious mathematical difficulties, however, and for the purpose of design calculations, therefore, we will simplify

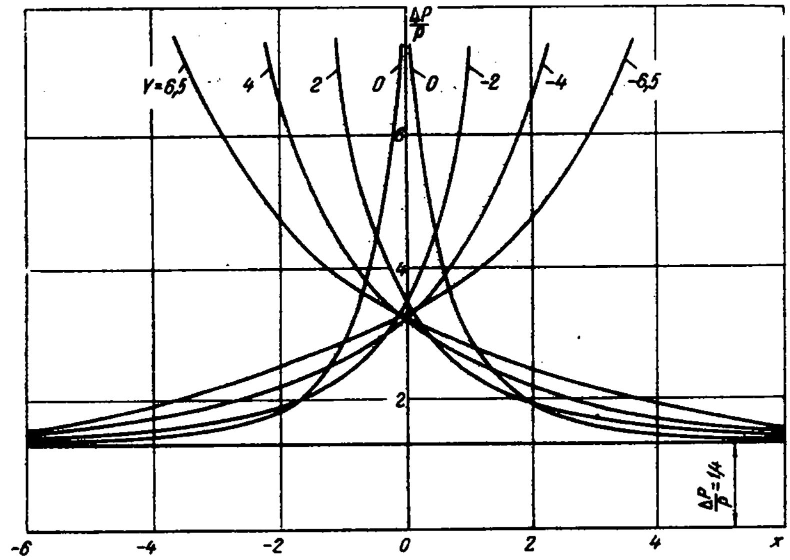

Fig. 2. Error $\Delta P/P$ as a function of the thermal state of the calorimeter.

the thermophysical and thus also the mathematical model of this calorimeter. First of all, we will assume that variations in the ambient temperature constitute the only source of extraneous noise and that there is no thermal flux q on the outside. We then modify the model as follows: the thermal capacity of the insulation C_i will be referred to the thermal capacity of the shell C_{sh} and the thermal resistance of the insulation R_i will be added to its outer thermal resistance $R_e = (\alpha S_i)^{-1}$, namely

$$C = C_i + C_{sh}, \quad R = R_i + R_e. \tag{8}$$

Thus, we have replaced a system of four media (chamber–gap–shell–insulation) by a system of three media (chamber–gap–effective shell) whose temperature field is described by the following equations:

$$P = C_{ch}\frac{dt_{ch}}{d\tau} + \sigma_1(t_{ch} - t_{sh}), \quad \sigma_1 = kS_{ch} \tag{9a}$$

$$\sigma_1(t_{ch} - t_{sh}) = C\frac{dt_{sh}}{d\tau} + \frac{1}{R}(t_{sh} - t_a). \tag{9b}$$

From Eq. (9a) we obtain

$$t_{sh} = t_{ch} + \frac{C_{ch}}{\sigma_1}\frac{dt_{ch}}{d\tau} - \frac{P}{\sigma_1}. \tag{10}$$

Substituting for t_{ch} its value from (10) into Eq. (9b), we have now

$$\frac{d^2t_{ch}}{d\tau^2} + d\frac{dt_{ch}}{d\tau} + gt_{ch} = f(\tau) + gt_a(\tau). \tag{11}$$

The notation here is as follows:

$$d = m_{sh} + m_{ch}(1+\beta), \quad g = m_{ch}m_{sh}, \quad m_{ch} = \frac{\sigma_{ch}}{C_{ch}},$$

$$f(\tau) = \frac{1}{C_{ch}}\left[\frac{dP}{d\tau} + (m_{sh} + \beta m_{ch})P\right], \quad m_{sh} = \frac{\sigma}{C},$$

$$\beta = \frac{C_{ch}}{C}, \quad \sigma = \frac{1}{R}.$$

The solution to Eq. (11) for $\lambda^2 = d^2 - 4g > 0$ is [5, 6]

$$t_{ch}(\tau) = A_1e^{-m_1\tau} + A_2e^{-m_2\tau} + \frac{2}{\lambda}\int_c^\tau \varphi(u)\,e^{\frac{d}{2}(u-\tau)}\,\mathrm{sh}\,\frac{\lambda}{2}(\tau - u)\,du,$$

$$m_1 = 0.5(d+\lambda), \quad m_2 = 0.5(d-\lambda), \quad \lambda = \sqrt{[m_{sh} + (1+\beta)m_{ch}]^2 - 4m_{ch}m_{sh}}, \tag{12}$$

$$\varphi(u) = f + gt_a.$$

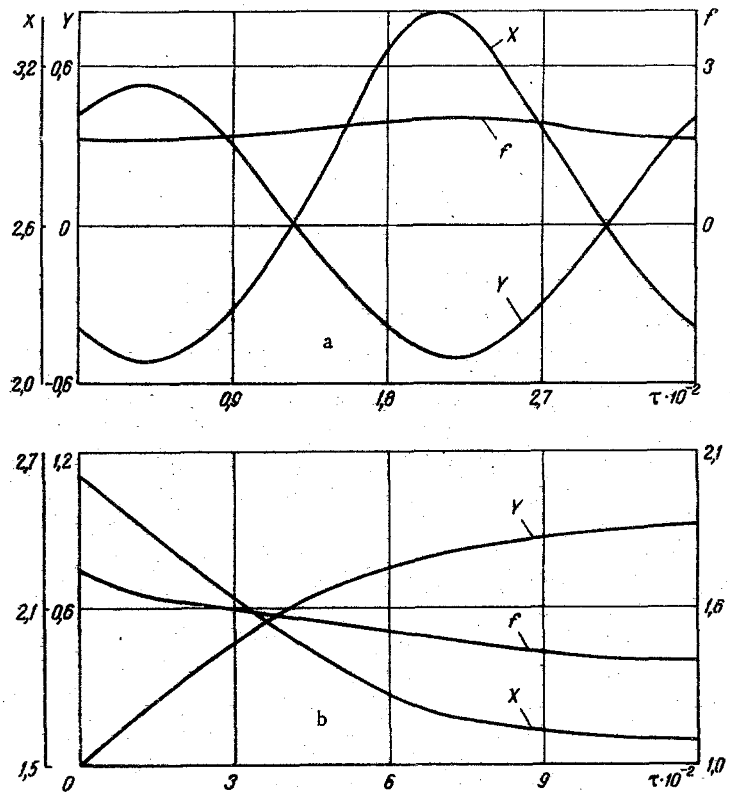

Fig. 3. Thermal state (X, Y) and error $f = \Delta P/P$ as functions of time: (a) during periodic variations in the ambient temperature (A = 20°K, T = 1 h), (b) after thermal shock ($\vartheta_a' = 10$°K).

If function $\varphi(u)$ is given, then Eqs. (10) and (12) yield relations $X(\tau)$ and $Y(\tau) = \Pi/m_{ch}$. In other words, when the power $P(\tau)$ and the ambient temperature $t_a(\tau)$ are known functions of time, then the thermal state of the device can be determined.

We will next derive expressions describing the thermal state of the device in two most practical cases, namely under a periodically varying ambient temperature and under a so-called thermal shock. We will also assume that the power of the source inside the chamber remains constant.

1. When the ambient temperature varies periodically

$$\vartheta_a = t_a - (t_a)_0 = A\cos\omega\tau, \quad \omega = 2\pi/T$$

then Eqs. (10) and (12) yield for a time long after the first instant

$$X = PR_{ch} + \frac{A\left[\frac{\omega}{m_2}\cos\beta_2\sin(\omega\tau-\beta_2) - \frac{\omega}{m_1}\cos\beta_1\sin(\omega\tau-\beta_1)\right]}{\sqrt{(1+R/R_{ch}+R/R_{ch}\beta)^2 - 4R/R_{ch}\beta}},$$

$$Y = \frac{A\left[\frac{\omega}{m_1}\cos\beta_1\sin(\omega\tau-\beta_1) - \frac{\omega}{m_2}\cos\beta_2\sin(\omega\tau-\beta_2)\right]}{\sqrt{(1+R/R_{ch}+R/R_{ch}\beta)^2 - 4R/R_{ch}\beta}}, \tag{13}$$

$$R_{ch} = \sigma_{ch}^{-1}, \quad \beta_1 = \mathrm{arctg}\,\frac{\omega}{m_1}, \quad \beta_2 = \mathrm{arctg}\,\frac{\omega}{m_2}.$$

2. Now we consider a thermal shock. Let the ambient temperature ϑ_a remain equal to zero during the time interval $0 \le \tau \le \tau_1$. At some instant of time $\tau = \tau_1$ the ambient temperature changes suddenly to ϑ_a'. The calorimeter then undergoes a thermal shock. This can be expressed analytically as follows:

$$\vartheta_a = \begin{cases} 0, & 0 \leqslant \tau \leqslant \tau_1, \\ \vartheta_a', & \tau \geqslant \tau_1. \end{cases}$$

At any instant of time the thermal state of the device can be described by the following expressions: at a time $0 \le \tau \le \tau_1$

$$X(\tau) = PR_\Sigma\left[\frac{\sigma}{\sigma+\sigma_{ch}} - \frac{m_{06}}{\lambda} e^{-\frac{d\tau}{2}} \operatorname{sh}\frac{\lambda}{2}\tau - \frac{m_1\sigma\left(e^{-m_1\tau} - \frac{m_2}{m_1}e^{-m_2\tau}\right)}{(m_1-m_2)(\sigma+\sigma_{ch})}\right],$$

$$Y(\tau) = PR_\Sigma\left[\frac{m_1\sigma}{(m_1-m_2)(\sigma+\sigma_{ch})}\left(e^{-m_1\tau} - \frac{m_2}{m}e^{-m_2\tau}\right) - \frac{m_{sh}}{\lambda} e^{-\frac{d\tau}{2}} \operatorname{sh}\frac{\lambda}{2}\tau\right].$$

Here $R_\Sigma = (\sigma + \sigma_{ch})/\sigma\sigma_{ch}$. At a time $\tau \ge \tau_1$

$$X(\tau) = \frac{P}{\sigma_{ch}} - \frac{m_2}{m_1-m_2}\left[\vartheta_{ch} - \vartheta_{sh}|_{\tau=\tau_1} - \frac{m_2}{m_1}\vartheta_{ch}|_{\tau=\tau_1} - \frac{P}{\sigma_{ch}}\right]\left[e^{-m_2(\tau-\tau_1)} - \frac{m_1}{m_2}e^{-m_1(\tau-\tau_1)}\right] + \frac{m_2}{m_{ch}}\vartheta_{ch}|_{\tau=\tau_1} e^{-m_2(\tau-\tau_1)}$$
$$- \frac{2m_{sh}}{\lambda}(PR_\Sigma + \vartheta'_a)e^{-\frac{d}{2}(\tau-\tau_1)} \operatorname{sh}\frac{\lambda}{2}(\tau-\tau_1),$$

$$Y(\tau) = \frac{m_2}{m_1-m_2}\left[\vartheta_{ch} - \vartheta_{sh}|_{\tau=\tau_1} - \frac{m_2}{m_{ch}}\vartheta_{ch}|_{\tau=\tau_1} - \frac{P}{\sigma_{ch}}\right]$$
$$\times\left[e^{-m_2(\tau-\tau_1)} - \frac{m_1}{m_2}e^{-m_1(\tau-\tau_1)}\right] - \frac{m_2}{m_{ch}}\vartheta_{ch}|_{\tau=\tau_1} e^{-m_2(\tau-\tau_1)} + \frac{2m_{sh}}{\lambda}(PR_\Sigma + \vartheta'_a)e^{-\frac{d}{2}(\tau-\tau_1)} \operatorname{sh}\frac{\lambda}{2}(\tau-\tau_1).$$

The dynamic calorimeter described in [7] had been developed at the Thermophysics Department of the Institute of Precision Mechanics and Optics in Leningrad for the study of heat transfer in warm-blooded animals, the results of design calculations are shown in Fig. 3a, b. Knowing the thermal state has also made it possible to select the parameters of the chamber, the shell, and the insulation for holding the error $\Delta P/P$ in power measurements within prescribed limits.

In conclusion, we note that the relations derived here are valid generally and can be used for calculating the thermal state of various devices to which the model in Fig. 1 applies.

APPENDIX

Equation (2) can be derived as follows. We know [4] that the mean-squared error ΔP in a power measurement is

$$\Delta P = \sqrt{\left(\frac{\partial f}{\partial A}\right)^2 \Delta A^2 + \left(\frac{\partial f}{\partial \Pi}\right)^2 \Delta \Pi^2 + \left(\frac{\partial f}{\partial B}\right)^2 \Delta B^2 + \left(\frac{\partial f}{\partial (t_{ch} - t_{sh})}\right)^2 \Delta (t_{ch} - t_{sh})^2}, \quad (14)$$

where $A = 1/mF$, $B = 1/F$, $\Pi = dt_{ch}/d\tau$, and function f is given in the form (1):

$$P = f(A, B, \Pi, t_{ch} - t_{sh}).$$

Dividing both sides of (14) by P from (1) yields an expression for the mean-squared error $\Delta P/P$. It then becomes necessary to calculate all the derivatives in (14). The relative errors $\Delta m/m$ and $\Delta F/F$ here are found by transforming the formulas

$$m = \frac{\ln(t_{ch} - t_{sh})_3 - \ln(t_{ch} - t_{sh})_4}{\tau_4 - \tau_3}, \quad F = \left(\frac{t_{ch} - t_{sh}}{P}\right)_c.$$

A proof of these two formulas is given in [7]. Algebraic transformations then yield formula (2).

NOTATION

P is the power generated by a source, W;
τ is the time, sec;
C_{ch} is the thermal capacity of the chamber, J/°K;
S_{ch} is the area of the outer surface of the chamber, m^2;
C_{sh} is the thermal capacity of the shell, J/°K;
S_{sh} is the area of the outer surface of the shell, m^2
k is the coefficient of heat transmission through the gap from the chamber to the shell, $W/m^2 \cdot {}^\circ K$;
t_{ch} is the chamber temperature, °K;
t_{sh} is the shell temperature, °K;

t_i	is the insulation temperature, °K;
t_a	is the ambient temperature, °K;
λ_i	is the thermal conductivity of the insulation, W/m·°K;
a_i	is the thermal diffusivity of the insulation, m^2/sec;
x	is the space coordinate, m;
α	is the coefficient of heat transfer at the interface between the insulation and the ambient medium, $W/m^2 \cdot °K$;
$q(\tau)$	is the thermal flux density, W/m^2;
X, Y	are the parameters which characterize the thermal state of a device, °K;
$\Pi = dt_{ch}/d\tau$	is the rate of change of the chamber temperature, °K/sec;
R_i	is the thermal resistance of the insulation, °K/W;
σ_{ch}	is the thermal conductance from chamber to shell, W/°K;
σ	is the thermal conductance from chamber to shell with the insulation taken into account, W/°K;
C	is the total thermal capacity of shell and insulation, J/°K;
A_1, A_2	are the integration constants, °K;
A	is the amplitude of fluctuations of the ambient temperature, °K;
T	is the period of fluctuations of the ambient temperature, sec;
R_Σ	is the total thermal resistance from the chamber to the ambient medium, °K/W;
$\vartheta_{ch} = t_{ch} - t_0$	is the superheat of the chamber above its initial temperature t_0, °K;
$\vartheta_{sh} = t_{sh} - t_0$	is the superheat of the shell above its initial temperature t_0, °K;
$\vartheta_a = t_a - t_0$	is the superheat of the ambient medium above its initial temperature t_0, °K;
$f = \Delta P/P$	is the error in a power measurement, %;
$\Delta t_{ch,i}, \Delta t_{sh,i}, \Delta\tau_i$	are the absolute errors in temperature and time measurements (i = 1, 2, 3, 4, c), °K and sec respectively;
$(\Delta P/P)_c$	is the error in the power calibration, %.

Subscripts

1, 2	denote temperatures during measurement;
3	denotes temperature in the steady mode;
4	denotes temperature in the transient mode;
c	denotes temperature during calibration.

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